**Math 231 – HW 5 Name: Troy Jeffery**

Epp 2nd Ed. 3.1 1ab, 2ab, 3, 6, 8, 9, 13, 14, 15, 16, 25, 27

***Remember -- FORMAT is as important as CONTENT – get them both right!***

3.1 (1) Assume that m and n are particular integers. Justify your answers to each of the following questions:

(a) Is 6m+8n even?

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| Proof: | Even if: n=2k  Let n = 6m+8n  6m+8n = 2k  3m+4n = k  So, 2(3m+4n) = 2k  Then 2(Integer) = Even | The sum or product of integers will always be an integer. |

(b) Is 10mn + 7 odd?

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| Proof: | Odd if: n=2k+1  Let n = 10mn + 7  10mn+7 = 2k+1  10mn+6 = 2k  5mn+3 = k  So, 2(5mn+3)+1 = 2k+1  Then 2(Integer)+1 = Odd | The sum or product of integers will always be an integer. |

3.1 (2) Assume that r and s are particular integers. Justify your answers to each of the following questions:

(a) Is 4rs even?

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| Proof: | Even if: n=2k  Let n = 4rs  4rs = 2k  2rs = k  So, 2(2rs) = 2k  2(Integer) = Even | The sum or product of integers will always be an integer. |

(b) Is 6r+4s2+3 odd?

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| Proof: | Odd if: n=2k+1  Let n = 6r+4s2+3  6r+4s2+3 = 2k+1  6r+4s2+2 = 2k  3r+2s2+1 = k  So, 2(3r+2s2+1) +1 = 2k+1  Then 2(Integer)+1 = Odd | The sum or product of integers will always be an integer. |

*Prove the statements in problems 3 and 6:*

3.1 (3) There is an integer n>5 such that 2n-1 is prime.

Proof: The number 6 is greater than 5 and 27 – 1 equals 127 which is prime.

3.1 (6) There is a real number x so that 2x > x10.

Proof: The number 0 is a real number

Let x = 0,

f(0) = 20 > 02 =>1 > 0.

*Prove the statements in 8 and 9 by the method of exhaustion:*

3.1 (8) Every positive even integer less than 26 can be expressed as a sum of three or fewer perfect squares. (For instance, 10 = 12 + 32, and 16 = 42.)

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| Theorem: | , n can be expressed as a sum of three or fewer perfect squares. | | |
| Proof (Exhaustion): | Even integers < 26:  (2, 4, 6, 8,10, 12, 14, 16, 18, 20, 22, 24) | 2 = 12 + 12  4 = 22  6 = 22 + 12 + 12  8 = 22 + 22  10 = 32 + 12  12 = 22 + 22 + 22 | 14 = 32 + 22 + 12  16 = 42  18 = 32 + 32  20 = 42 + 22  22 = 32 + 32 + 22  24 = 42 + 22 + 22 |

Also: What's the first positive even integer for which the statement is NOT true?

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| 28 is the first integer that cannot be expressed as a sum of three or fewer perfect squares. | 25 + 4 > 28  25 +1 + 1 < 28 | 16 +16 > 28  16 + 9 + 9 > 28  16 + 9 + 4 > 28  16 + 9 + 1 < 28 | 9 + 9 + 9 < 28 |

3.1 (9) For each integer n such that 1 ≤ n ≤ 10, n2 - n + 11 is a prime number.

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| Theorem: | 1 ≤ n ≤ 10, n2 - n + 11 is a prime number. | |
| Proof (Exhaustion): | n(1) = 1 - 1 + 11 = 11 (1, 11)  n(2) = 4 – 2 + 11 = 13 (1, 13)  n(3) = 9 – 3 + 11 = 17 (1, 17)  n(4) = 16 – 4 + 11 = 23 (1, 23)  n(5) = 25 – 5 + 11 = 31 (1, 31) | n(6) = 36 – 6 + 11 = 41 (1, 41)  n(7) = 49 – 7 + 11 = 53 (1, 53)  n(8) = 64 – 8 + 11 = 67 (1, 67)  n(9) = 81 – 9 + 11 = 83 (1,83)  n(10) = 100 – 10 + 11 = 101 (1,101) |

Also: Why did the theorem stop at n = 10?

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| n(11) = 121 – 11 + 11 = 121 (1, 11, 121) | At n(11) the output is 121 which isn’t prime. |

*Prove the statements in problems 13 and 14. Follow the directions for writing proofs of universal statements given in this section.*

3.1 (13) If n is any even integer, then (-1)n = 1.

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| Theorem: | , (-1)n = 1 |
| Proof: | If even: n=2k  Let k = any integer  (-1)2k=1  ((-1)2)k = 1  1k=1 |

3.1 (14) If n is any odd integer, then (-1)n = -1.

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| Theorem: | , (-1)n = -1. |
| Proof: | If odd: n = 2k + 1  Let k = any integer  (-1)2k+1 = -1  (-1)2k + (-1)1 = -1  ((-1)2)k -1 = -1  1k \* (-1) = -1 |

*Disprove the statements in problems 15 and 16 by giving a counterexample. Answer with a complete sentence!*

3.1 (15) For all positive integers n, if n is prime, then n is odd.

False: 2 is a positive integer that is prime, but not odd.

3.1 (16) For all real numbers a and b, if a < b, then a2 < b2.

False: If a = -2, and b = 1 then there is a condition where

a < b, but the square of a is not less than the square of b.

*Prove the statements that are true, and give counterexamples to disprove the statements that are false:*

3.1 (25) The product of any two odd integers is odd.

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| Theorem: | , x\*y is odd. |
| Proof: | If odd: n=2k+1  x = 2c+1  y = 2b+1  (2c+1)(2b+1) = 4bc + 2b + 2c + 1  So, 2(2bc +b + c) +1 = 2(k) + 1  Then 2(Integer) + 1 is odd. |

3.1 (27) The difference of any two odd integers is odd.

False: a = 3, b = 1, If you subtract b from a you will get two, which is not odd.